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First-order transitions in fluctuating 1+1-dimensional nonequilibrium systems

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We demonstrate that first-order phase transitions in 1+1-dimensional nonequilibrium systems with fluctuating ordered phases are impossible, provided that there are no additional conservation laws, long-range interactions, macroscopic currents, or special boundary conditions. Since minority islands in the ordered phase of such systems can only shrink by short-range interactions, it is impossible to stabilize a fluctuating ordered state. The apparent first-order behavior turns out to be a transient phenomenon, crossing over to a continuous transition after very long time. As examples we consider the triplet creation model $3X \to 4X$, $X \to \emptyset$, the annihilation/fission process $2X \to 3X$, $2X \to \emptyset$, as well as spreading on a diffusing background $X+Y \to 2Y$, $Y \to X$.

I. INTRODUCTION

In equilibrium statistical mechanics it is well known that systems undergoing a first-order phase transition in high space dimensions d may exhibit a second-order transition below their upper critical dimension. The reason is that in low dimensions fluctuations become more important and may destabilize the ordered phase. A very similar situation emerges in the case of nonequilibrium phase transitions [1]. In this context the question arises under which conditions first-order phase transitions can be observed in one spatial dimension. The purpose of the paper is to point out that various 1+1-dimensional nonequilibrium models, which were believed to exhibit a first-order transition, cross over to a continuous transition after very long time.

Dynamic random processes with first-order phase transitions are usually characterized by several stable ordered phases. For example, in the subcritical regime $0 < T < T_c$ of the two-dimensional kinetic Ising model there are two stable magnetized states. To ensure their stability, the Ising model provides a robust mechanism eliminating islands of the minority phase generated by thermal fluctuations. This mechanism relies on the fact that the boundary of an island costs energy, leading to an effective *surface tension*. Attempting to minimize its energy, the island is subjected to an attracting 'force' and begins to shrink. It is important to note that this long-range force decreases *algebraically* as $1/r$ with the typical radius r of the island so that thermal fluctuations of any size are safely eliminated. Because of the Z_2-symmetry under spin reversal, both ordered phases are equally attracting. Thus, starting from a disordered state with zero magnetization, we observe coarsening patterns of ordered domains. However, if an external field is applied, one type of minority islands becomes unstable above a certain critical size. Since there is a finite probability to generate such islands by fluctuations, one of the two ordered phases eventually takes over, i.e., the system undergoes a first-order phase transition.

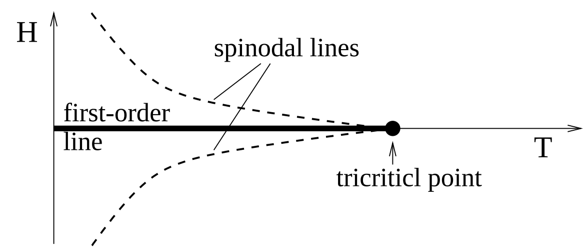

FIG. 1. Schematic phase diagram of Toom's north-east-center voting model.

Turning to nonequilibrium systems, the mechanism for the elimination of minority islands may be even more robust. An interesting example is Toom's two-dimensional north-east-center voting model [2,3]. Again there are two stable ordered phases. In contrast to the kinetic Ising model, where straight domain walls perform an unbiased diffusive motion, the interfaces in Toom's model propagate in a preferred direction, wherefore the process is out of equilibrium. As a key property of the model, this propagation velocity depends on the orientation of the domain walls. Using this anisotropy, the dynamic rules of the model are designed in a way that minority islands quickly assume a triangular shape and begin to shrink with constant velocity. Thus, the effective 'force' by which an island shrinks is independent of r, leading to much more stable phases as in the standard kinetic Ising model. Consequently, the ordered phases remain stable even if an oppositely oriented external field H is applied. In order to flip the whole system, the intensity of the field has to exceed a certain critical threshold. The corresponding phase stability boundaries (also called spinodal lines) are sketched in Fig. 1. Between these lines, the two ordered states are both thermodynamically stable, i.e., they coexist in a whole region of the parameter space. Crossing the coexistence region by varying the

external field, the magnetization of the system follows in a hysteresis loop. Similar phenomena can be observed in certain models for nonequilibrium wetting [4].

Obviously, both mechanisms for the elimination of minority islands – surface tension and anisotropic propagation velocities – can only be implemented in at least two spatial dimensions. Therefore, it is interesting to investigate the question under which conditions first-order phase transitions can be observed in one spatial dimension. For example, the 1+1-dimensional Ziff-Gulari-Barshad model [5] for heterogeneous catalysis is known to exhibit a first-order phase transition which relies on the interplay of three different kinds of particles. Similarly, a recently introduced model for phase separation on a ring [6] uses three different species of particles. Another example is the so-called bridge model [7] for bidirectional traffic on a single lane, where special boundary conditions induce a discontinuous transition in the currents. Even more subtle is the mechanism in the two-species model introduced in a recent paper by Oerding *et al.*, where a first-order phase transition is induced by fluctuations [8]. Therefore, in attempting to comprehend the full range of first-order phase transition under nonequilibrium conditions, it would be interesting to seek for the simplest 1+1-dimensional model which exhibits a discontinuous transition. By 'simple' we mean that such a model should involve only one species of particles evolving by local dynamic rules without macroscopic currents, conservation laws, and unconventional symmetries. Moreover, the choice of the boundary conditions should be irrelevant.

The prototype of such a dynamic process is the one-dimensional Glauber-Ising model at zero temperature in a magnetic field. This model, which is also referred to as *compact directed percolation* (CDP), can be defined as follows. Sites of a one-dimensional lattice can be in two different states $\uparrow$ and $\downarrow$. For each update a pair of adjacent sites is randomly selected. If the two spins are in opposite states, the are aligned with the probabilities p and $1-p$:

$$\uparrow\downarrow, \downarrow\uparrow \xrightarrow{p} \uparrow\uparrow\,, \qquad \uparrow\downarrow, \downarrow\uparrow \xrightarrow{1-p} \downarrow\downarrow\;. \tag{1}$$

Obviously, the parameter $H = p-1/2$ plays the role of an external field. Since there is no temperature, the model has two *absorbing configurations*, namely the fully magnetized states $A = ...\downarrow\downarrow\downarrow...$ and $B = ...\uparrow\uparrow\uparrow...$. Once the system has reached one of the two absorbing configurations, it is trapped and will remain there forever.

In the off-critical case $H \neq 0$, one of the two absorbing states is stable while the other one is unstable against small perturbations. For example, for $H < 0$, B-islands tend to grow while A-islands shrink. Thus, starting with random initial conditions, the system approaches the thermodynamically stable state B in an exponentially short time. In the vicinity of the phase transition this time scale diverges as $|H|^{-1}$. Right at the critical point, the two absorbing states of the Glauber model are only *marginally* stable against perturbations. For example, by flipping a single spin in a fully magnetized domain, we create a pair of kinks. These kinks perform an unbiased random walk until they annihilate one another. Thus, minority islands do not shrink by virtue of an attracting force, rather they are eliminated solely because of the fact that random walkers in one dimension always return to their origin. Consequently, the lifetimes τ of minority islands are finite and distributed algebraically as $P(\tau) \sim \tau^{-1/2}$. It is important to note that this mechanism allows islands to reach a macroscopic size of the order $\sqrt{\tau}$ during their temporal evolution. Therefore, averaging over many independent samples, the mean size of such minority islands approaches a constant value. As a consequence, the ordered phase is only marginally stable against perturbations. In fact, introducing a small rate for spontaneous spin flips, the first-order transition in the Glauber-Ising model is lost. The same happens in certain nonequilibrium Ising models with two absorbing states subjected to an external field [9,10].

Is it possible to observe first-order transitions in *fluctuating* 1+1-dimensional two-state systems? Obviously, such a model requires a much more robust mechanism for the elimination of minority islands. A simple random walk of a pair of kinks is not sufficient, rather there has to be an attracting force which prevents small island from growing. It seems that such a mechanism is difficult to implement. In fact, various models, which have been suggested in the past, display a discontinuous transition only in $d \geq 2$ dimensions [11–14,9,15].

A frequently cited exception is the 1+1-dimensional triplet creation process introduced by Dickman and Tomé [16]:

$$3X \to 4X\,, \quad X \to \emptyset\,. \tag{2}$$

In this model the high-density phase is not strictly absorbing, rather islands of unoccupied sites are spontaneously created in the bulk so that the active state fluctuates. In numerical simulations it was observed that above a certain tricritical point the second-order phase transition line splits up into two spinodal lines, where the transition becomes first order. Moreover, the order parameter seemed to follow a hysteresis loop when the parameter for offspring production was varied. Apparently, the highly nonlinear particle creation process $3X \to 4X$ is able to stabilize the high-density phase, eliminating islands of the minority phase. Similarly, Carlon *et al.* [17] reported a first order transition in the 1+1-dimensional annihilation/fission process on a lattice [18]

$$2X \to 3X\,, \quad 2X \to \emptyset\,. \tag{3}$$

For low values of the diffusion constant they found a continuous transitions which becomes first order for high diffusion rates above a certain tricritical point.

In the present work we argue that this type of first-order behavior in 1+1-dimensional models is only a tran-

sient phenomenon. Although mean field approximations predict a discontinuous behavior, the transition crosses over to a continuous transition for any value of the parameters (i.e., for any value of the diffusion constant in the examples mentioned before).

The paper is organized as follows. Discussing the dynamics of interacting domain walls in Sect. II, we argue that short-range forces are not sufficient to stabilize fluctuating ordered phases in one spatial dimension. In Secs. III and IV we revisit the triplet creation process and the annihilation/fission process, both being examples of models with short-range forces. As a third example a spreading process on a diffusing background will be discussed in Sec. V. Although in this case long-range forces lead to a coarsening process, the transition turns out to be continuous as well.

II. INTERACTING DOMAIN WALLS IN ONE DIMENSION

Short-range forces between domain walls

To explain the impossibility of first-order transitions in the processes (2)-(3) from a phenomenological point of view, let us consider a hypothetical one-dimensional system with two ordered phases **A** and **B**. Without loss of generality we assume the **A**-phase to be absorbing while the **B**-phase fluctuates, i.e. small **A**-islands are spontaneously created in the bulk of the **B**-phase. Furthermore, we assume that there is a robust mechanism which eliminates minority islands generated by fluctuations, ensuring the stability of phase **B**. For a system with these properties, let us consider an initial state where half of the system is in phase **A** while the other half is in phase **B** (see Fig. 2). Both phases are separated by a domain wall. In contrast to the Glauber-Ising model, this domain wall is not necessarily associated with a single broken bond, rather is may be 'smeared out' over a certain region in space. However, since both phases are attracting, the domain wall remains *localized*, i.e., it has a finite width. Consequently, the derivative of the order parameter profile has exponentially decreasing tails. Obviously, the critical point of such a model corresponds to a situation where the domain wall diffuses without bias.

In one dimension a minority islands of phase **A** inside phase **B** can be considered as a pair of such domain walls. In the Glauber-Ising model the domain walls do not interact unless they annihilate at the same site. In our hypothetical model, however, the domain walls do interact, giving rise to an attracting 'force' eliminating islands of the minority phase. However, since the domain walls have a finite width, we expect this 'force' to have a finite range r_0. More precisely, for large island sizes r we expect the strength of the interaction to decrease *exponentially* as $\exp(-r/r_0)$ or even faster.

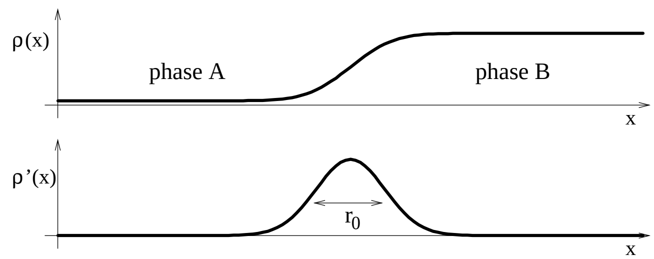

FIG. 2. Schematic profile of the coarse-grained particle density $\rho(x)$ and its derivative $\rho'(x)$ at a domain wall between phase **A** and phase **B** in a one-dimensional system.

Instability of fluctuating ordered domains

The essential problem arises precisely at this point. In one spatial dimension an exponentially decreasing short-range force between two domain walls is not sufficient to stabilize a fluctuating ordered phase. In order to understand this point, let us consider an island of size r which grows by one step with rate $1 - a\exp(-r/r_0)$ and shrinks with rate $1+a\exp(-r/r_0)$. In this toy model the parameters a and r_0 control the strength and the range of the force, respectively. The time evolution of the corresponding probability distribution $P_r(t)$ is given by the equation*

$$\frac{\partial}{\partial t}P_r(t) = -2P_r(t) + (1 + a\,e^{-(r+1)/r_0})P_{r+1}(t) \qquad (4)$$
$$+(1 - a\,e^{-(r-1)/r_0})P_{r-1}(t)$$

with the boundary condition $P_0(t) = 0$. Starting with a single spin flip $P_r(0) = \delta_{r,1}$, we obtain a broadening distribution of island sizes. Without a short range force (i.e. $r_0 = 0$), Eq. (4) reduces to the discrete diffusion equation with a Dirichlet boundary condition at $r = 0$. In this case the asymptotic solution reads

$$P_r(t) \overset{t\to\infty}{\simeq} C\,t^{-3/2}\exp(-r^2/4t)\,, \qquad (5)$$

where C is a certain amplitude factor. In presence of a short-range force with $r_0 > 0$, this asymptotic solution remains valid – the only change is in the prefactor C. Although C decreases exponentially with increasing r_0, it is always positive. Therefore, there is a small but finite probability that the two domain walls become effectively unbound, producing a macroscopic minority island with a size of the order $\sqrt{\tau}$. Thus, the ordered phase is only marginally stable, irrespective of the interaction range r_0. As a consequence, the **B**-phase eventually disintegrates, approaching the absorbing state **A**. This contradiction in our hypothetical model demonstrates that in one spatial dimension *it is impossible to stabilize fluctuating ordered*

*In principle there may be two different length scales on both sides of the domain wall.

phases by short-range interactions. It should be emphasized that these arguments are not valid in higher dimensions where the dynamics of domain walls may depend on the local curvature.

Properties of the continuous transition
Since absorbing islands may reach a macroscopic size, our hypothetical model approaches the absorbing state. In order to prevent the system from reaching the absorbing state, we may now slightly modify the parameters such that **B** domains tend to grow. Since the **A**-phase is absorbing, the resulting competition between growing **B**-domains and spontaneously created **A**-islands suggest a crossover to directed percolation (DP) [19,20], which is the generic universality class of continuous phase transition into absorbing states. For instance, as will be shown in Sec. III, the triplet creation process displays such a crossover to DP. In general, the universality class of the continuous transition will depend on the symmetry properties and the conservation laws of the process under consideration. As an example, we will discuss the annihilation/fission (see Sec. IV) which crosses over to a non-DP transition.

Numerical checks
Obviously these considerations are only valid if the ordered states are sufficiently attracting to confine domain walls to a finite region with a typical size r_0. There are several possibilities to verify this assumption. The simplest method would be to perform high-precision Monte Carlo simulations in order to confirm the crossover to a continuous transition. Alternatively, we may check whether a compact B-phase disintegrates as predicted. As a more sensitive check, one could also measure the distribution of the sizes of minority islands. Since spin flips occur spontaneously in the ordered phase, we expect this probability distribution to become stationary in the limit $t \to \infty$. Solving the stationary problem of Eq. (4), we obtain the distribution

$$P_r = \prod_{r'=1}^{r-1} \frac{1 - a\, e^{-r'/r_0}}{1 + a\, e^{-(r'+1)/r_0}}\, P_1 \,. \qquad (6)$$

This distribution decays exponentially for small r until it saturates at a small but finite constant in the limit $r \to \infty$, expressing the fact that minority islands of all sizes will be formed. A distribution of this form observed in a numerical simulation can be considered as a hallmark for the presence of a short-range force between domain walls.

III. THE TRIPLET CREATION PROCESS $3X \to 4X$, $X \to \emptyset$

We are now going to apply these tests to the triplet creation model introduced by Dickman and Tomé [16]. It evolves by random sequential updates and is controlled by two parameters, namely a rate for offspring production λ and a diffusion constant D. For $D < 0.85$ the transition was found to belong to the universality class of directed percolation, whereas for $D > 0.85$ a first order transition was reported. Moreover, the phase transition line seemed to split up into two spinodal lines $\lambda_\pm(D)$. In the following we show that the transition crosses over to DP after very long time. We restrict the analysis to the case $D = 0.9$, where Dickman and Tomé reported the stability limits $\lambda_- = 10.12(1)$ and $\lambda_+ = 10.30(2)$.

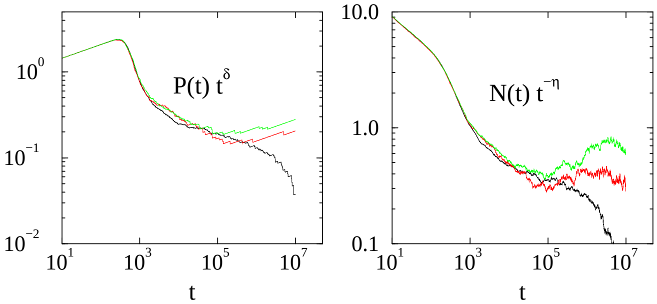

FIG. 3. Numerical simulation of the triplet creation process. Starting with an island of 20 particles the survival probability $P(t)$ (left) and the mean number of particles $N(t)$ (right) of the cluster are averaged over 300 independent runs. The data points are multiplied with powers of t in a way that DP critical behavior corresponds to horizontal lines (see text).

As a first test, we perform Monte Carlo simulations starting with a single island of particles at the origin [21]. It turns out that it is necessary to start with an island of several particles since otherwise the survival probability of the cluster is extremely small. At criticality the survival probability $P(t)$ and the average number of particles $N(t)$ are expected to obey asymptotic power laws of the form

$$P(t) \sim t^{-\delta}\,, \qquad N(t) \sim t^{\eta} \qquad (7)$$

with $\delta = 1/2, \eta = 0$ for CDP (indicating a first-order transition) and $\delta \simeq 0.159, \eta \simeq 0.313$ in the case of a DP transition, respectively. The results of the simulations are shown in Fig. 3. Obviously, there are three different temporal regimes. In the first 100 time steps, the island survives with certainty due to the large initial size of 20 sites, followed by a rapid decrease of $P(t)$ faster than $1/\sqrt{t}$. This part of the temporal evolution is expected to be non-universal. Then the model enters a second regime extending over two decades from 10^3 to 10^5 time steps, where $P(t)$ decays as $1/\sqrt{t}$ and $N(t)$ stays almost constant. This is the time window where the model behaves essentially in the same way as a zero-temperature Glauber-Ising model so that the transition appears to be discontinuous. In the following two decades from 10^5 to 10^7 time steps, however, we observe a slow crossover to DP exponents. Our estimate for the critical point $\lambda = 10.145(10)$ lies between the stability limits $\lambda_\pm$ reported in [16]. With the computer technology ten years

ago, Dickman and Tomé could only go up to 22000 time steps so that they were unable to observe the crossover to DP. A similar crossover phenomenon from an initial transient over CDP to DP was recently observed in certain models for flowing sand on an inclined plane [22].

As a second check, we demonstrate that the fluctuating phase disintegrates, generating the typical patterns of DP clusters after very long time. To this end we introduce a novel type of space-time plots which can be used to visualize the scaling properties of critical clusters in systems with absorbing states (see Fig. 4). Starting with a localized seed (an island of 20 particles) at the origin and simulating the process up to 10^7 time steps, the rescaled position of the particles $x/t^{1/z}$ is plotted against $\log_{10} t$, where $z = \nu_\|/\nu_\perp$ is the dynamic exponent of the process under consideration. Here we choose $z = 2$ since the domain walls are expected to diffuse. By rescaling the spatial coordinate x, the cluster evolves within a strip of finite width. Unlike linear space-time plots the scale-invariant representation of a cluster allows one to survey more than six decades of the temporal evolution. As can be seen, the figure is consistent with the Monte Carlo simulation results. After an initial transient the cluster appears to be compact, behaving essentially in the same way as in a Glauber-Ising model at zero temperature. The boundaries, which are smeared out by diffusion, become sharper as time proceeds. The first minority island with a macroscopic size appears after $2 \cdot 10^4$ time steps, where the crossover to DP begins. However, the typical patterns of a DP cluster can only be seen after 10^6 time steps.

As a third check, we measure the stationary distribution P_r of island sizes in order to verify the predictions of the toy model in Sect. II. Starting with a fully occupied lattice, the sizes of inactive islands are measured in a critical system of 10000 sites after an 'equilibration' time of 10^6 time steps. The result is shown in Fig. 5 as a solid line. As can be seen, the distribution P_r decays exponentially or even faster by almost five decades until it crosses over to a slowly decreasing function. Fitting the parameters a and r_0, this curve may be compared with the stationary solution of the toy model in Eq. (6). Although the curves do not coincide, they show the same qualitative behavior. This supports the point of view that domain walls in the 1+1-dimensional triplet creation process attract one another by a short-range force so that the fluctuating active phase cannot be stabilized.

IV. THE ANNIHILATION/FISSION PROCESS $2X \to 3X$, $2X \to \emptyset$

Some time ago Howard and Täuber introduced the so-called annihilation/fission process which exhibits an unconventional nonequilibrium phase transition [18]. Their original motivation was to consider a reaction-diffusion

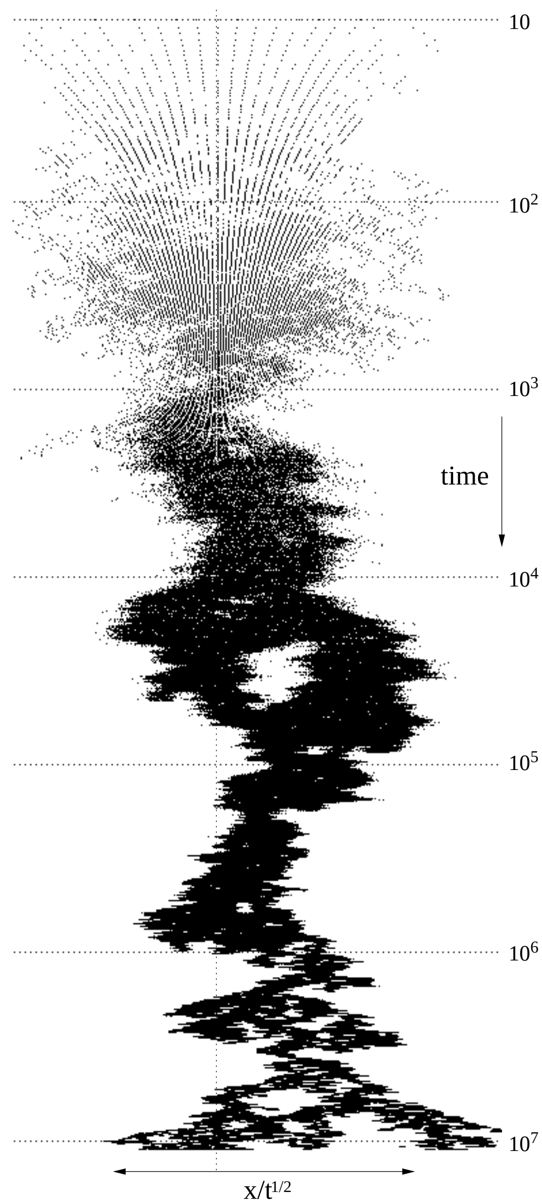

FIG. 4. The triplet creation process $3X \to 4X$, $X \to \emptyset$ at the critical point. The figure shows the temporal evolution over seven decades starting with a compact island of 20 particles in the center. The graphs shows the rescaled position $x/\sqrt{t}$ of the particles as a function of $\log(t)$ measured in a single run. Up to 10^4 time steps the cluster has the form of a compact diffusing cloud of particles with very small islands of unoccupied sites generated by fluctuations. The first macroscopic minority island emerges after $2 \cdot 10^4$ time steps, indicating the beginning of the crossover to DP which extends up to 10^6 steps. Only in the last decade, where the thickness of active branches is small compared to the lateral cluster size, we observe the typical patterns of a directed percolation process.

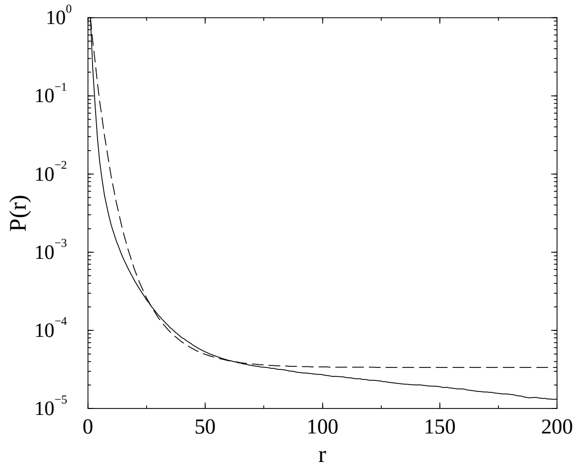

FIG. 5. Distribution of the sizes of inactive islands in the critical triplet creation model (solid line), compared to a fit of Eq. (6) (dashed line).

process which allows one to interpolate between 'real' and 'imaginary' noise in the corresponding Langevin equation. Performing a field-theoretic renormalization group analysis they predicted non-DP critical behavior at the transition.

More recently, Carlon *et al.* [17] studied a lattice version of the model which is defined by the dynamic rules

$$\begin{aligned} AA\emptyset,\, \emptyset AA \to AAA \qquad & \text{with rate}\,(1-p)(1-d)/2 \\ AA \to \emptyset\emptyset \qquad & \text{with rate}\,p(1-d) \\ A\emptyset \leftrightarrow \emptyset A \qquad & \text{with rate}\,D\ . \end{aligned} \tag{8}$$

Performing a density matrix renormalization group analysis they arrived at the conclusions that for low values of the diffusion constant D the transition is continuous while it becomes first order for higher values of D above a certain tricritical point. The paper by Carlon *et al.* released a debate concerning the universality class of the continuous phase transition. Because of a numerical coincidence of two out of four critical exponents, the authors concluded that the transition should belong to the parity-conserving universality class [23–28]. This result was questioned in Ref. [29] since there is neither a parity conservation law nor a Z_2-symmetry in the AF process. Moreover it was pointed out that a first order transition might not exist in one spatial dimension. Increasing the diffusion constant, continuously varying critical exponents were reported in [30]. For large values of D these exponents seemed to approach certain values which can also observed in cyclically coupled DP and annihilation processes [31]. So far there is no numerical evidence for a discontinuous transition.

It should be emphasized that the AF process has two non-symmetric absorbing states, namely the empty lattice and the state with a single diffusing particle. The unusual critical behavior is related to the fact that solitary particles may diffuse over large distances before they meet another particle in order to annihilate or to release a new avalanche. Therefore, the absorbing state may be 'less attracting' than in the triplet creation model. Because of these special properties, it is not immediately clear whether the assumptions of Sect. II apply. For example, solitary diffusing particles could lead to an effective long-range interaction between domain walls. However, as demonstrated in Fig. 6, even for high values of the diffusion rate a compact island quickly disintegrates, generating nontrivial patterns of active patches and solitary particles. This supports the claim by Ódor [30] that there is no first-order phase transition in the AF process for any $0 < D < 1$. The question whether the observed continuously varying critical exponents are related to asymptotically well-defined properties or simply to crossover effects between an an effective first-order behavior on short scales and an asymptotic continuous transition remains open.

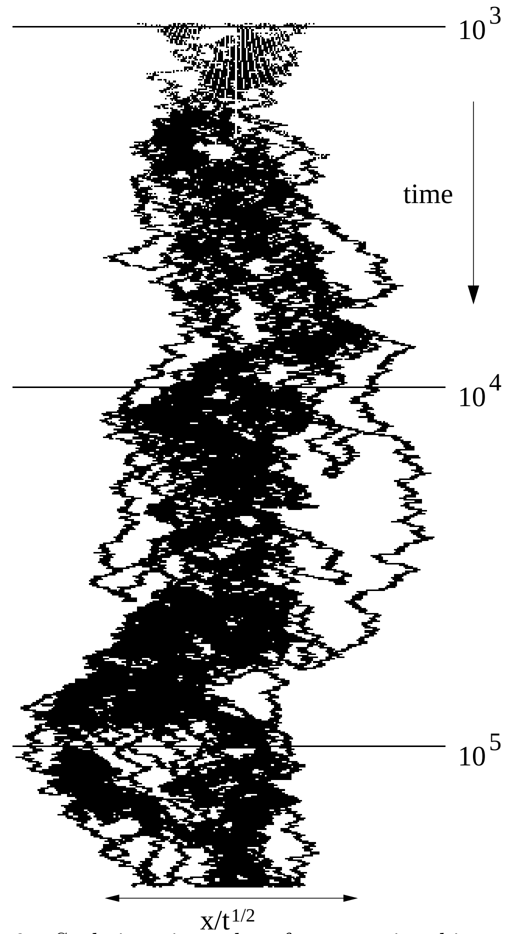

FIG. 6. Scale-invariant plot of a space-time history of the annihilation/fission process for $D = 0.9$ at the critical point $p_c = 0.233(2)$. As can be seen, the high density phase disintegrates, creating macroscopic islands of the minority phase in the bulk.

V. SPREADING PROCESS ON A DIFFUSING BACKGROUND

Let us finally investigate the following two-species particle process with random-sequential updates:

$$
\begin{array}{lll}
X+Y \to Y+Y & \text{with rate} & \alpha \\
Y \to X & \text{with rate} & \beta \\
X+\emptyset \leftrightarrow \emptyset+X & \text{with rate} & D_X \\
Y+\emptyset \leftrightarrow \emptyset+Y & \text{with rate} & D_Y
\end{array}
\qquad (9)
$$

This reaction diffusion scheme may be interpreted as a spreading process of Y particles on a diffusing background of X particles. In this process the total number of particles is conserved. Therefore, the spreading properties are controlled by the density of particles in the initial state $\rho = \rho_X + \rho_Y$, while the density ρ_Y plays the role of an order parameter. For low values of ρ the particles are sparsely distributed so that the system quickly evolves into an 'absorbing' state without Y particles. For high values of ρ the spreading process $X+Y \to 2Y$ dominates, leading to a stationary density $\rho_Y > 0$ in a sufficiently large system. Both phases are separated by a nonequilibrium phase transition at a certain critical density ρ_c which depends on the parameters α, β and the diffusion rates D_X and D_Y.

The critical behavior of the model depends significantly on the ratio of the diffusion rates D_X and D_Y. If both rates are equal, the particles diffuse in the same way as in an ordinary exclusion process, whereas the labels X and Y can be considered as 'colors' with separate dynamic rules. This special case has been considered in Ref. [32], where a continuous phase transition was found. Recently, van Wijland *et al.* were able to confirm these results by a field-theoretic analysis [33]. Moreover, they investigated the general case $D_X \neq D_Y$. For $D_X < D_Y$ they predicted a continuous phase transition with non-DP critical exponents, while for $D_X > D_Y$ it turned out to be impossible to perform a controlled field-theoretic analysis. For this case van Wijland *et al.* conjectured that the transition might be discontinuous.

Obviously, the arguments used in Sect. II cannot be applied to this model since the particle conservation law introduces effective long-range interactions. Thus, the model may well be a candidate for a first-order transition even in one spatial dimension. To investigate this question in more detail, we analyzed the two-species model numerically. Choosing $\alpha = 0.2$, $\beta = 1$ and $D_Y = 1 - D_X$, the process (9) is controlled by two parameters $D_X = D$ and ρ. The Y particles may be considered as the active sites of a spreading process running on top of a diffusing background of X particles. For $D \leq 0.5$ the transition is characterized by fractal clusters reminding of DP, but apparently in a different universality class. For $D > 0.5$, however, there is indeed a clear signature of first-order behavior. As shown in the space-time plot in Fig. 7, the Y particles are confined to certain vertical 'channels'. As time proceeds, small channels disappear while larger ones tend to grow, leading to a coarsening process.

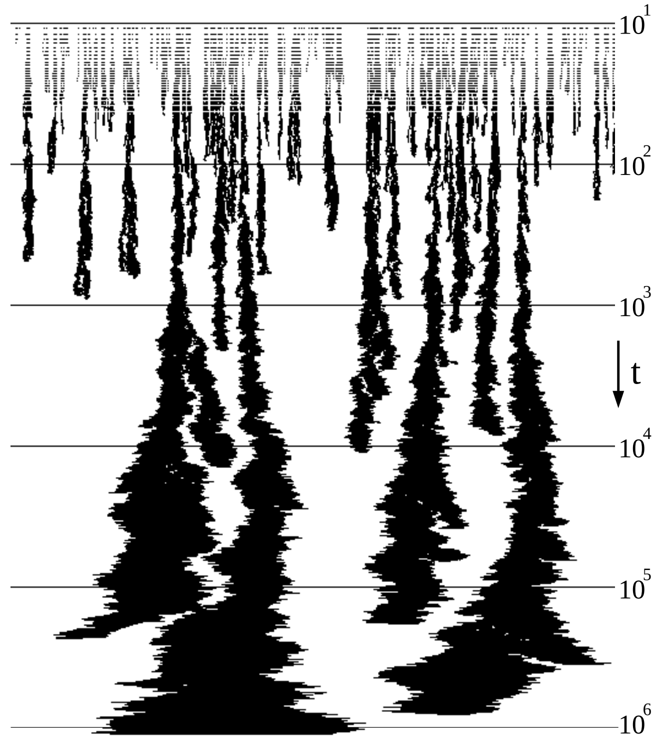

FIG. 7. Semi-logarithmic plot of the spreading process (9) with 1000 sites for $D = 0.9$ in the active phase $\rho - \rho_c$. The figure shows the dynamics of the Y particles, while the diffusing background of X particles is not visible. As can be seen, islands of Y particles coarsen until a single island is left.

The mechanism behind this coarsening process can be understood as follows. Because of $D_Y < D_X$, a high concentration of Y particles reduces the effective diffusion rate. Since a locally reduced diffusion rate leads to particle clogging, the local density of particles increases. Thus there is positive feedback amplifying the particle density (and hence the effective spreading rate) in regions where the spreading process is active. Because of this instability, the Y particles tend to form compact domains with a high particle density. In $d \geq 2$ spatial dimensions, such an instability may lead to a first-order transition. In one-dimensional systems, however, the nonlinear amplification mechanism alone is not sufficient, especially when the high-density phase fluctuates. Obviously, the essential mechanism behind the coarsening process is the conservation of the total number of particles. For example, let us assume that a high-density domain has been created by fluctuations. Because of the instability, the spreading process is supercritical inside the domain. Particle conservation ensures that such an island cannot split up into two separate islands since there is no way to reduce the density inside.

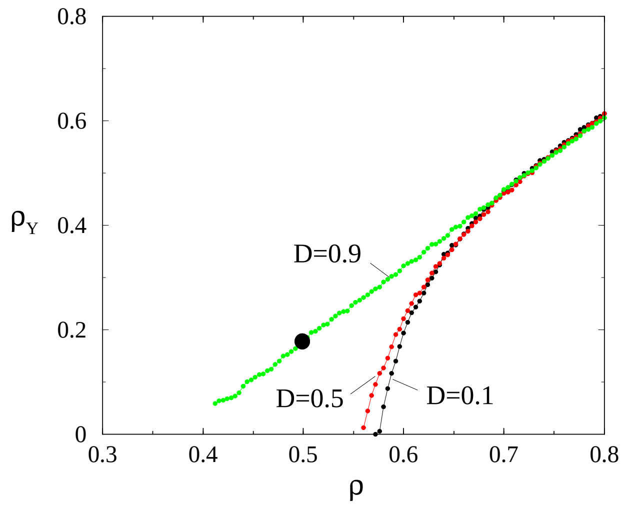

FIG. 8. Spreading process on a diffusing background. Stationary density of Y-particles as a function of the total particle density ρ for various values of the diffusion rate $D = D_x = 1 - D_Y$. The bold dot marks the location where the space-time plot in Fig. 7 has been generated.

The temporal evolution of the coarsening process depends on the dynamics of domain walls between regions with low and high particle density. Obviously, high-density islands tend to grow until the particle density of their environment decreases below a certain threshold. This threshold, however, depends not only on the initial particle density ρ but also on the total size of all active islands. In other words, active islands continue to grow until they bind so many particles that the average density of particles in their environment is no longer sufficient to sustain the growth process. Moreover, large islands seem to be more stable than small ones, leading to a slow coarsening process. Thus, finite systems evolve into a state where only one high-density domain survives.

The conservation law has another surprising consequence: Although the system coarsens, the transition is still continuous. In fact, plotting the asymptotic stationary value of ρ_B against $\rho - \rho_c$, there is no discontinuity (see Fig. 8). This can be explained by observing that the size of the surviving active domain grows almost linearly with $\rho - \rho_c$. Therefore, the reaction-diffusion (9) provides an interesting example of a system with an instability which exhibits a second-order phase transition in one dimension, although the arguments Sect. II cannot be applied.

VI. CONCLUSIONS

First-order transitions in 1+1-dimensional nonequilibrium models with fluctuating ordered phases require a robust mechanism which eliminates islands of the minority phase generated by fluctuations. In many cases this mechanism relies on special properties such as the interplay of several species of particles, competing currents, unconventional conservation laws, or special boundary conditions. In this paper we have presented physical arguments why first-order phase transitions are impossible in simple two-state models without such attributes since short-range interactions between domain walls cannot stabilize a fluctuating ordered phase. As an example we have revisited the triplet-creation model which was believed to display a first-order phase transition. However, the first-order behavior is a transient phenomenon and eventually crosses over to a continuous phase transition, supporting the DP conjecture by Jannsen and Grassberger [34,35]. Similarly, the annihilation/fission process crosses over to a second-order phase transition. Thus, first-order phase transitions in one spatial dimension are subtle and require a much more robust mechanism for the elimination of minority islands.

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